5-Gen: A Framework for Prototyping Applications Using Multilinear Maps and Branching Programs

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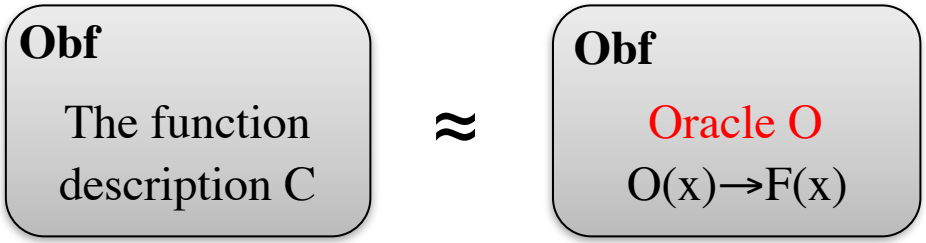
Obfuscation

- Make programs "unintelligible" while maintaining their functiona
- Hiding secrets in software?
  - Proprietary algorithms
  - Software patches
  - Malware detection

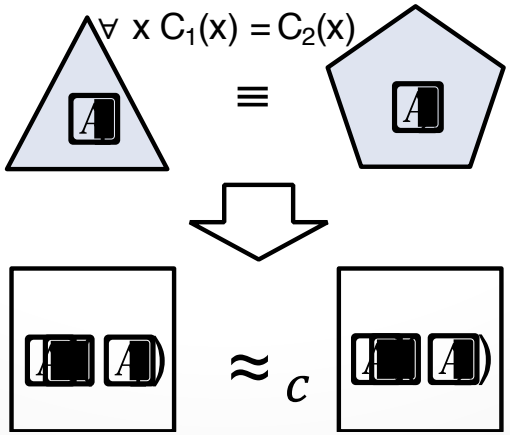
```
@P=split//,".URRUU
\c8R";@d=split//,"\nrekcah xinU / lreP
rehtona tsuJ";sub p{ @p{"r$p","u$p"}
=(P,P);pipe"r$p","u$p";++$p;($q*=2)+=$f=!
fork;map{$P=$P[$f^ord ($p{$_})&6];
$p{$_}=/ ^$P/ix?$P:close$_}keys%p}
p;p;p;p;p;map{$p{$_}=~/^[P.]/&& close$_}
%p;wait until$?;map{/^r/&&<$_>}%p;$_=
$d[$q];sleep rand(2)if/\S/;print
```

Obfuscation Security [BGIRSVY01]

- Virtual Black Box (VBB) security
  - Impossibility



- Indistinguishability Obfuscation



First candidate construction
[GGHRSW13]
and many others after

Functional Encryption (FE) [SW05]

- Decryption keys that reveal only functions of the encrypted message

Encryption

```
=(P,P);pipe"r$p","u$p";++$p;
($q*=2)+=$f=!fork;map{$P=
$P[$f^ord ($p{$_})&6];$p{$_}=/ ^
$P/ix?$P:close$_}keys%p}
p;p;p;p;p;map{$p{$_}=~/^[P.]/&&
close$_}%p;wait until$?;map{/^r/
&&<$_>}%p;$_=$d[$q];sleep
rand(2)if/\S/;print
```

Is the age over 21?

Driving License Category ?

"Flavors" of Functional Encryption

- Setup – generate parameters
  - Encryption key PK
  - Master secret key MSK

Private Key
$PK = MSK$

- KeyGen – generate function decryption keys
  - SK_F

Function Hiding
SK_F hides F

- Enc – generate ciphertext for a message
  - ct = Enc(PK, m)

Multi-Input FE (MIFE)
$ct_i = Enc(PK_i, x_i)$

- Dec – decrypt a ciphertext
  - $Dec(SK_F, ct) = F(m)$

Multi-Input FE (MIFE)
$Dec(SK_F, ct_1, \ldots, ct_n) = F(x_1, \ldots, x_n)$

Obfuscation and FE

- Function hiding public key FE ⇒ obfuscation

Obf(F)

SK_F

$Eval_{Obf(F)}(m)$

1. $ct = Enc(PK\ m)$
2. $Dec(SK_F, ct)$

- Multi-input Functional Encryption ⇒ obfuscation [AJ15]
  - n+1-slot MIFE ⇒ n-bit input obfuscation

Obf(F)

SK_U

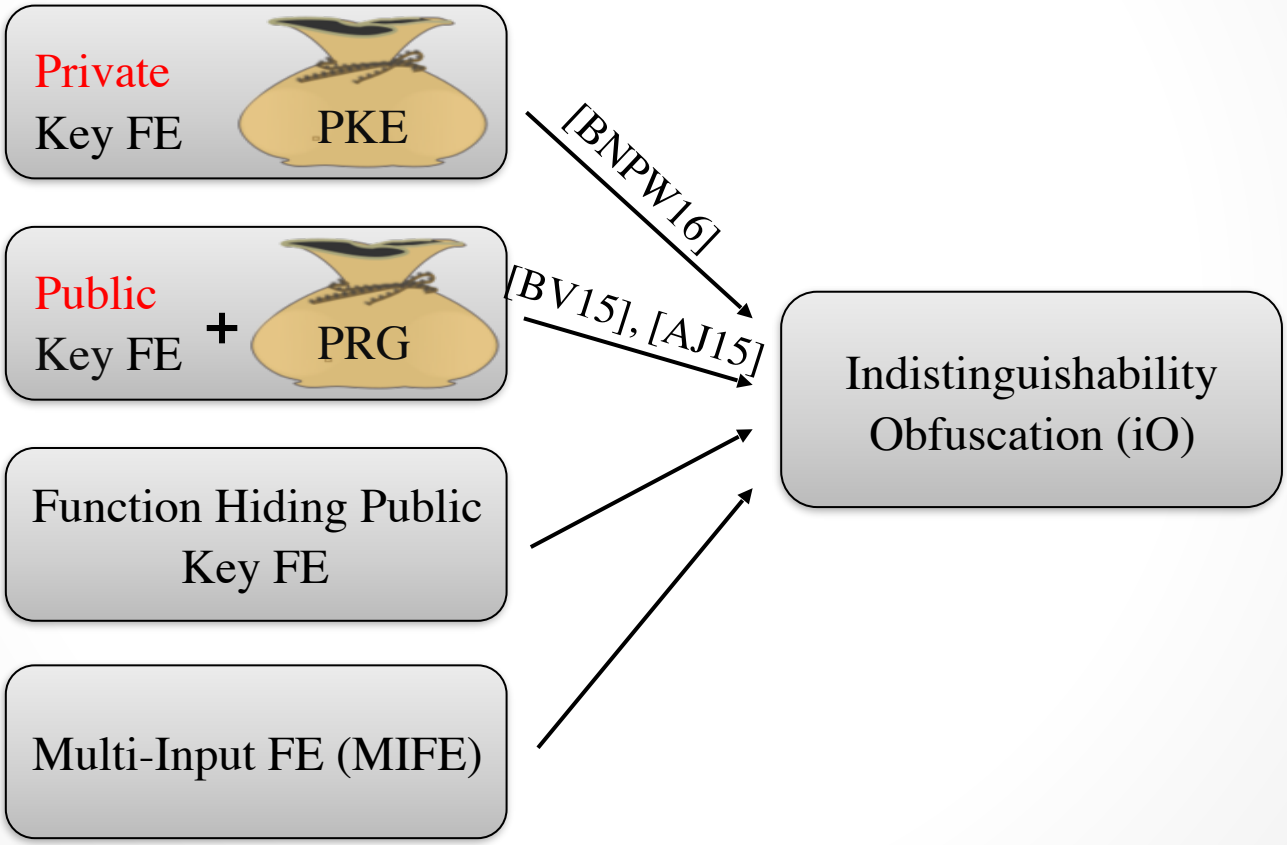
$Enc^{(0)}(F)$ $Enc^{(1)}(0)$ … $Enc^{(n)}(0)$

$Enc^{(1)}(1)$ … $Enc^{(n)}(1)$

$Eval_{Obf(F)}(m_1 \ldots m_n)$

$Dec\left(SK_u, Enc^{(0)}(F), Enc^{(1)}(m_1) \ldots Enc^{(n)}(m_n)\right)$

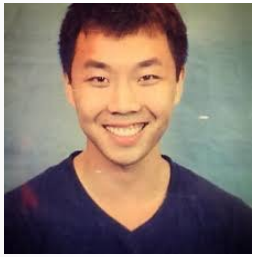
Obfuscation and FE

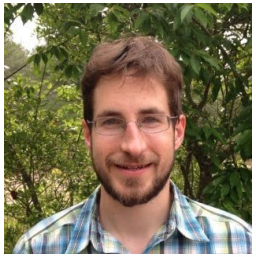


MMAPs Efficiency

Can we implement any of this?

FHE Efficiency

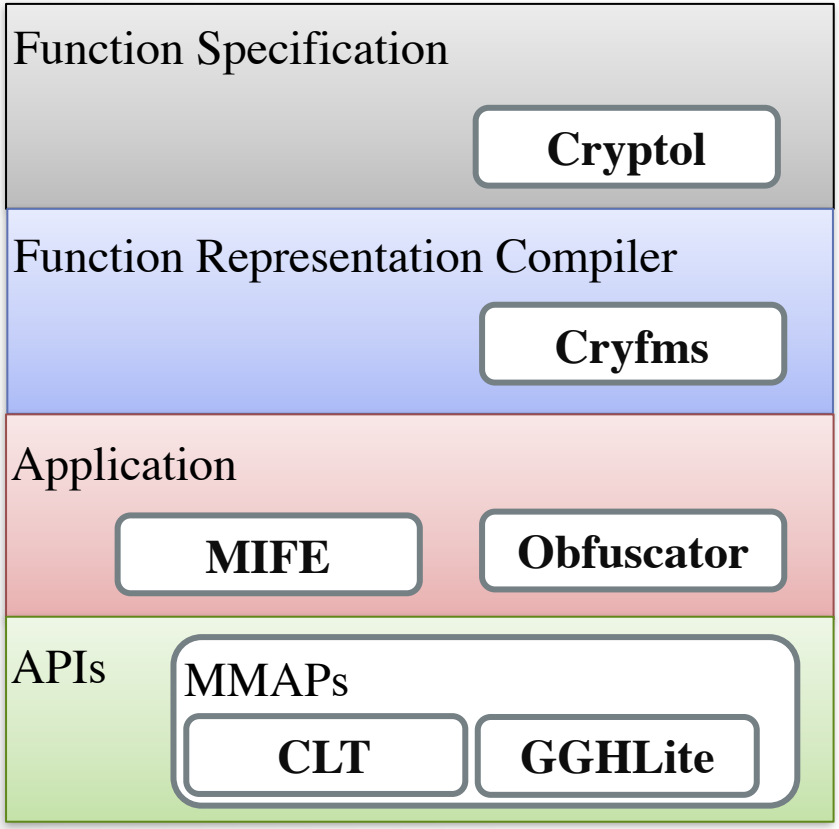


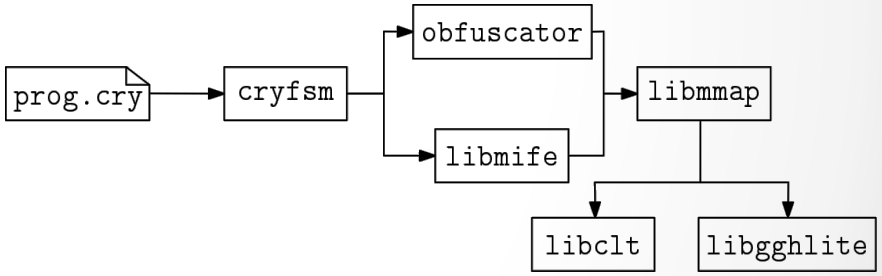


Summary of Results

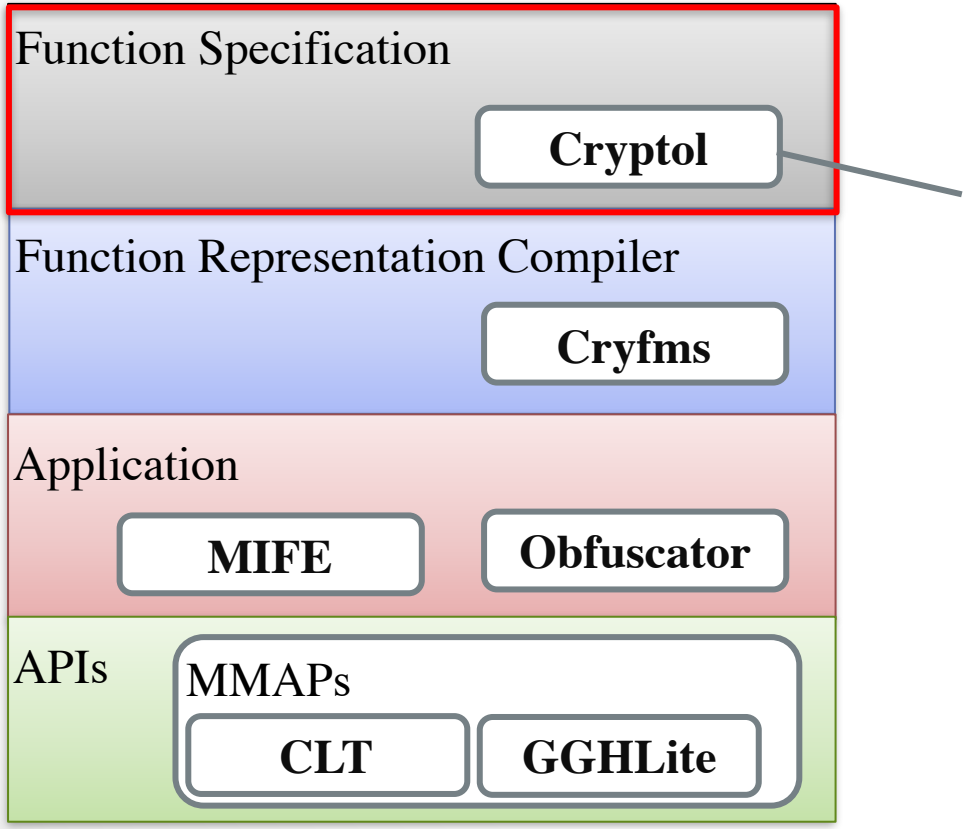
- **A framework for prototyping applications using multilinear maps and branching programs**
- Compiler from Cryptol to matrix branching programs (MBPs)
- API with multilinear map implementations
  - GGH Lite
  - CLT13 (no public encoding)
- Applications
  - Multi-input functional encryption (order revealing encryption (ORE))
  - Point function obfuscation
- Efficiency Evaluation

5Gen Architecture





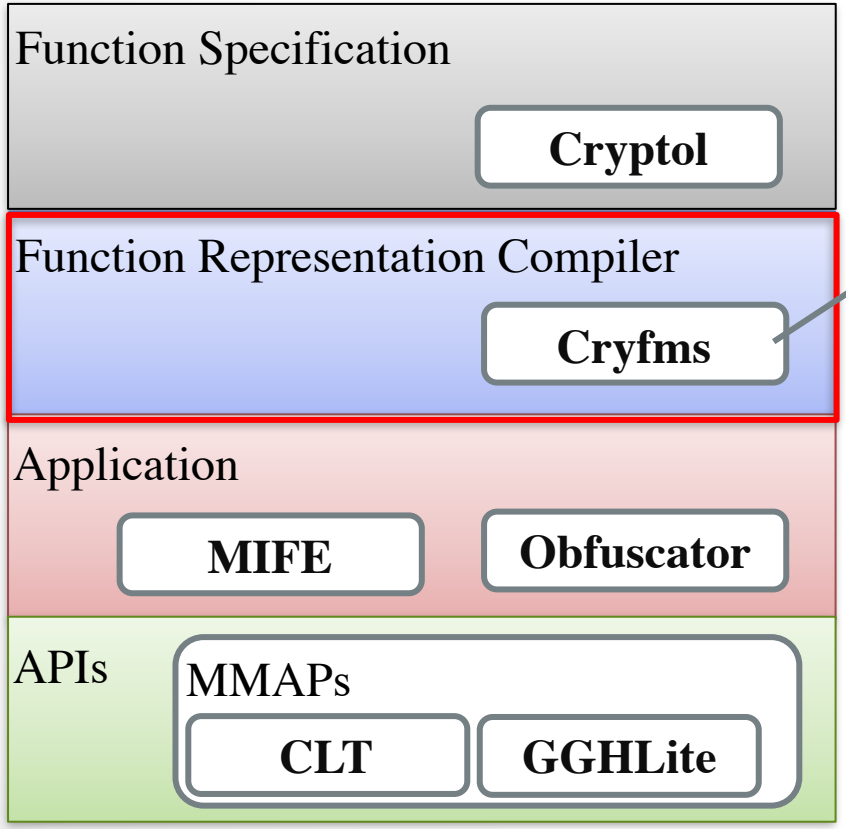
Function Representation



Language for describing cryptographic algorithms

- Executable format
- Formal property proofs for specification
- Prove equivalence with or generate implementations in other languages

Compiler



Compiler from Cryptol to minimal finite state machine and then MBP

- FSM optimizations – track reachable states
- MBP oprimizations

Branching Programs [B86]

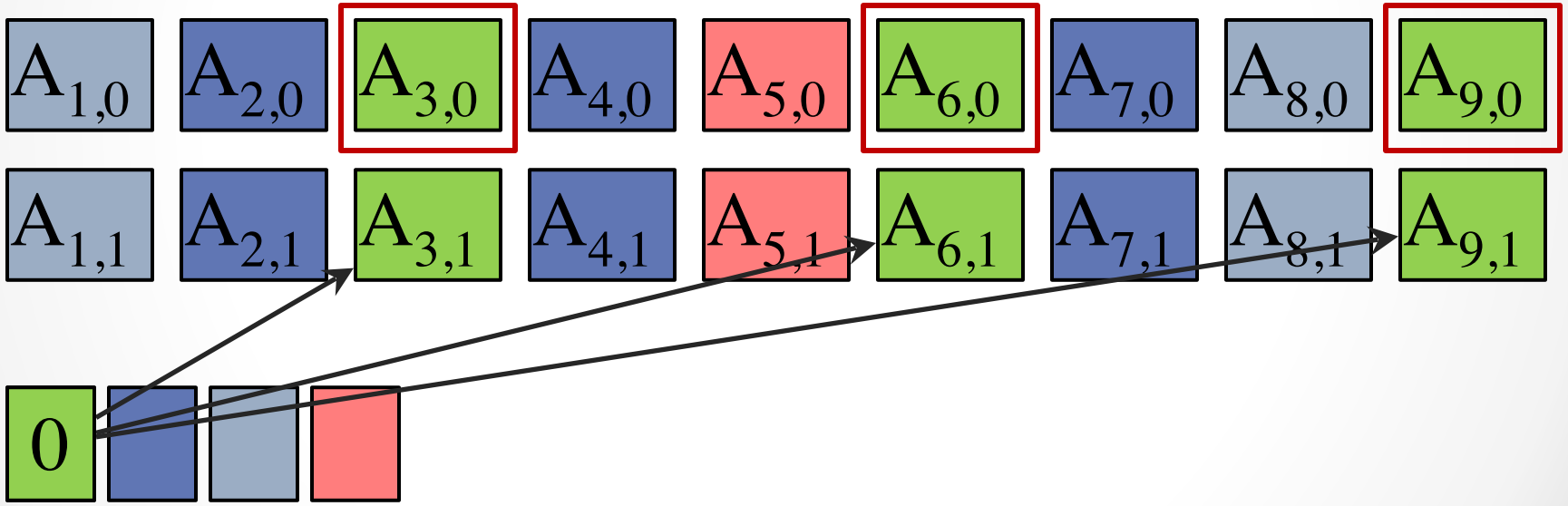
- BP of length *m* with *n* input bits is defined as
  $(inp(1), A_{1,0}, A_{1,1}), (inp(2), A_{2,0}, A_{2,1}), \ldots, (inp(m), A_{m,0}, A_{m,1})$
  - $A_{i,0}, A_{i,1} \in \{0,1\}^{k \times k}$
  - $inp(x) : [m] \rightarrow [n]$

- BP for *F* evaluates on input $x = (x_1, \ldots, x_n)$

$$F(x) = \begin{cases} 1, & \text{if } \prod_{i=1}^{n} A_{i,inp(i)} = I \\ 0, & \text{otherwise.} \end{cases}$$

Branching Programs

- Example: BP of length 9 with 4-bit inputs



Branching Programs

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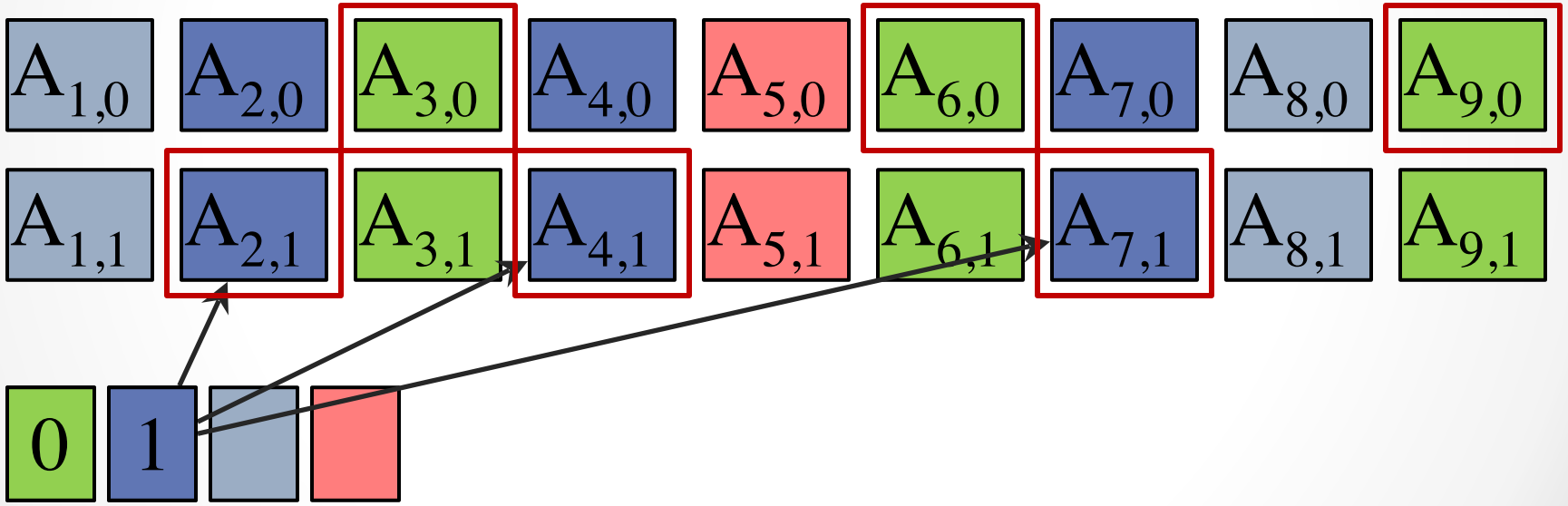
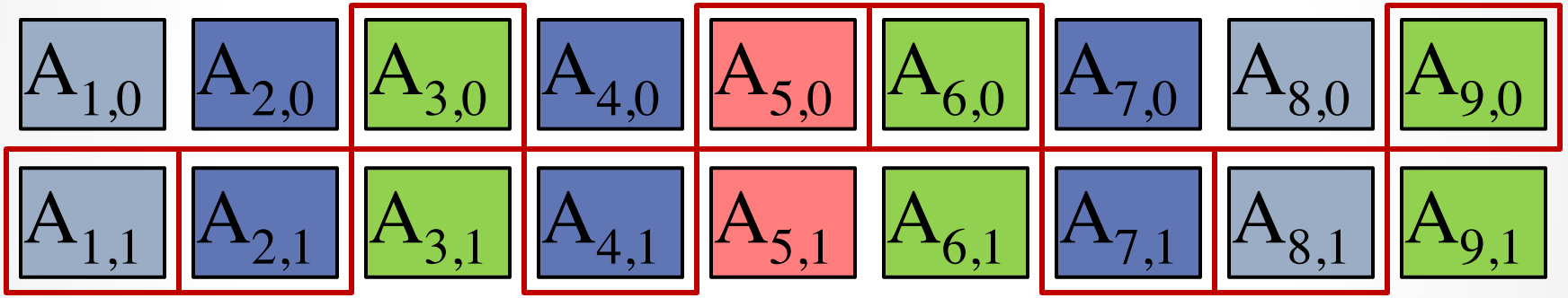
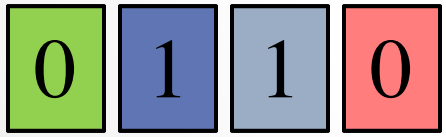


Branching Programs

- Example: BP of length 9 with 4-bit inputs



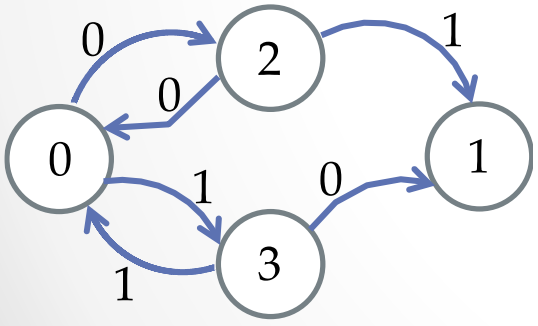


- Multiply the chosen 9 matrices.
  - If the product is I, output 1. Otherwise, output 0.

How to construct MBP?

- Barrington's construction: circuit to MBP
  - Size of the MBP is exponential in the circuit depth

- Finite State Automaton to MBP

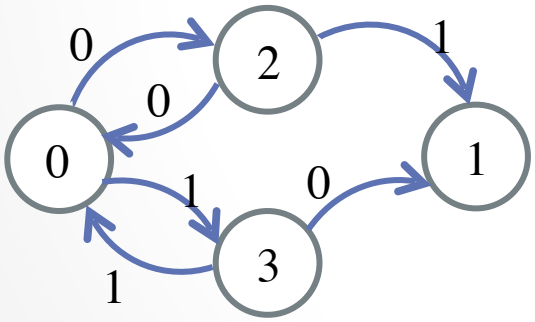


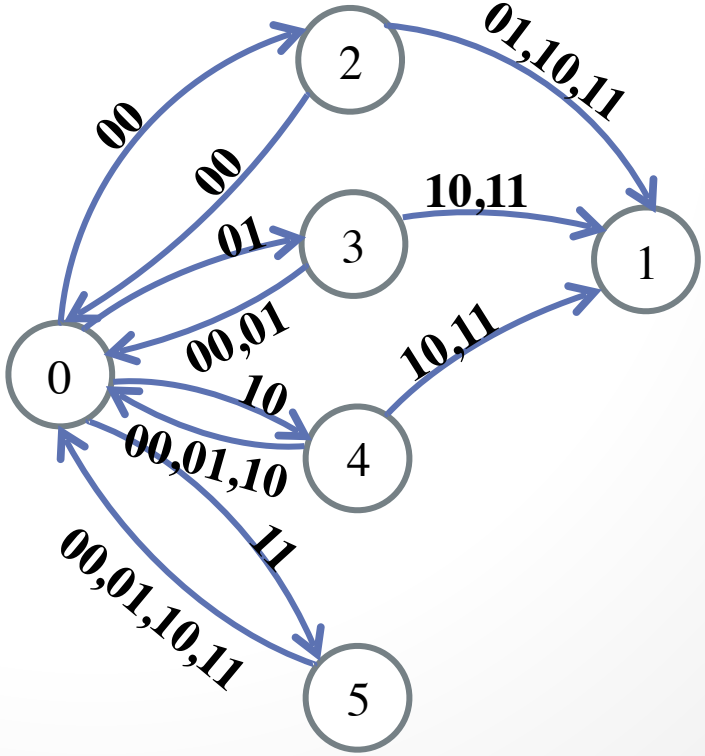
$$A_{1,0} = \begin{array}{c} \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 \\ \mathbf{1} & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & \mathbf{1} \end{pmatrix} \end{array} \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \qquad A_{2,0} = \begin{pmatrix} \mathbf{1} & 0 & \mathbf{1} & 0 \\ 0 & \mathbf{1} & 0 & \mathbf{1} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_{1,1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 \\ 0 & 0 & \mathbf{1} & 0 \\ \mathbf{1} & 0 & 0 & \mathbf{1} \end{pmatrix} \qquad A_{2,1} = \begin{pmatrix} \mathbf{1} & 0 & 0 & \mathbf{1} \\ 0 & \mathbf{1} & \mathbf{1} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Optimizations for MBP

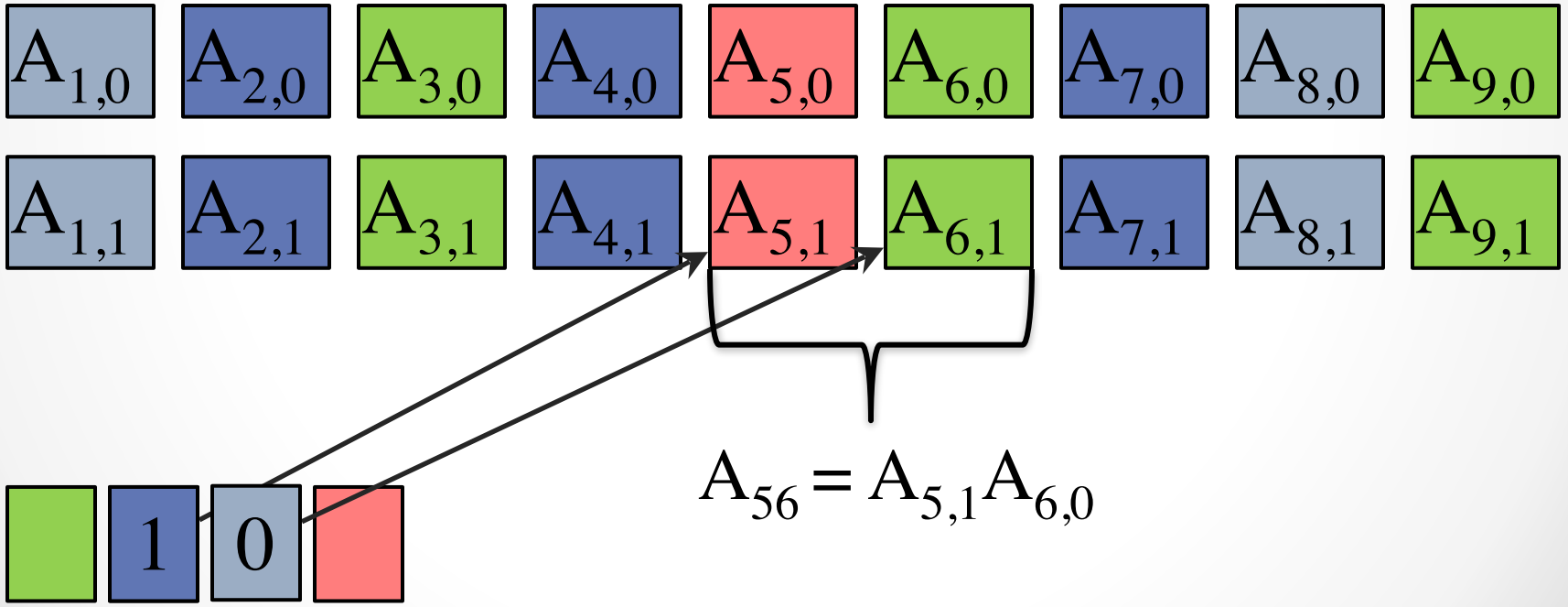
- Condensing the input representation
  - larger transition symbols alphabets (e.g. digits in larger base) – fewer transitions, more states





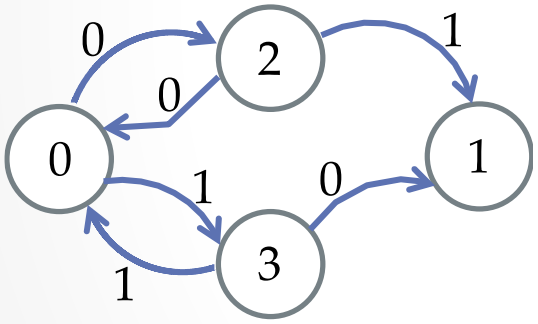
Optimizations for MBP

- Matrix premultiplication



Optimizations for MBP

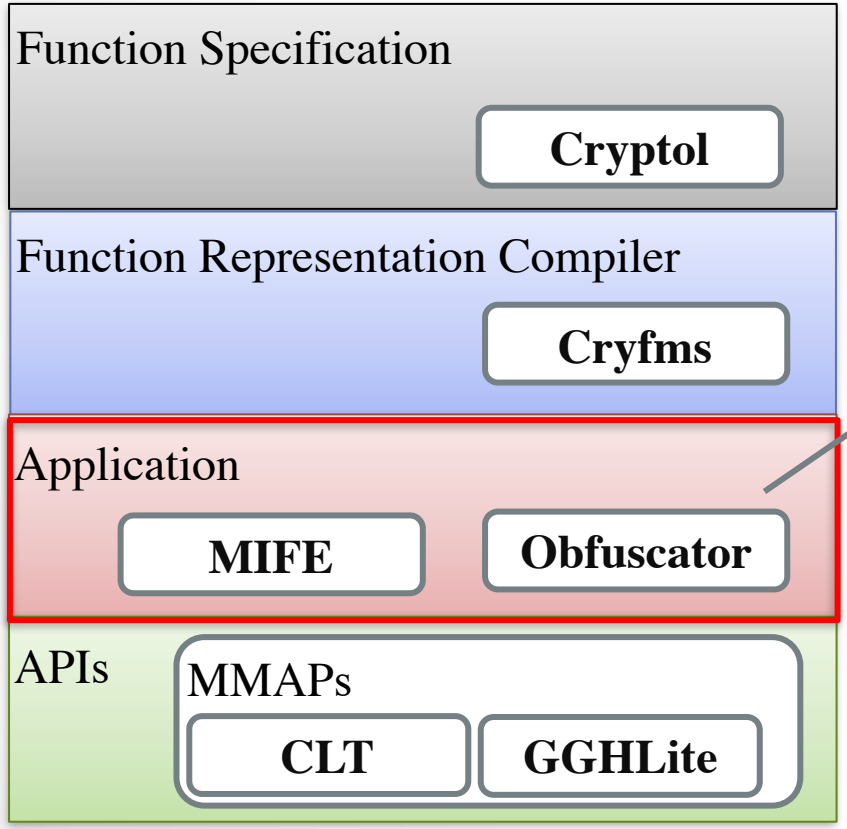
- Dimension reducing – for some bit positions the FSM transitions affect only some of the states



$$A_{1,0} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 \\ \mathbf{1} & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & \mathbf{1} \end{pmatrix} \qquad A_{2,0} = \begin{pmatrix} \mathbf{1} & 0 & \mathbf{1} & 0 \\ 0 & \mathbf{1} & 0 & \mathbf{1} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

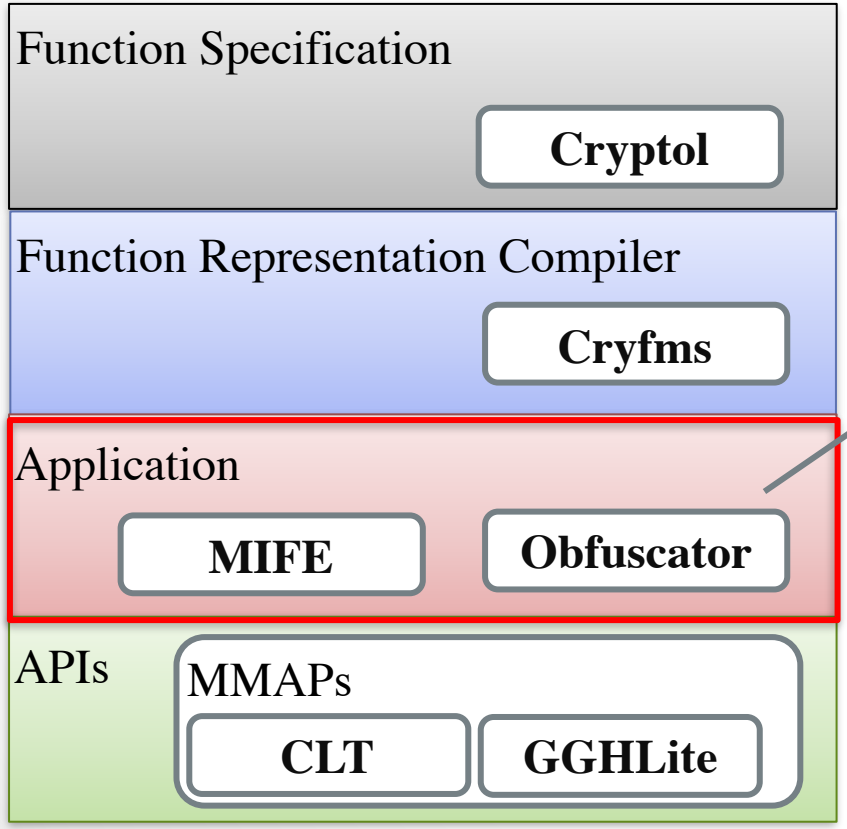
$$A_{1,1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 \\ 0 & 0 & \mathbf{1} & 0 \\ \mathbf{1} & 0 & 0 & \mathbf{1} \end{pmatrix} \qquad A_{2,1} = \begin{pmatrix} \mathbf{1} & 0 & 0 & \mathbf{1} \\ 0 & \mathbf{1} & \mathbf{1} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Applications



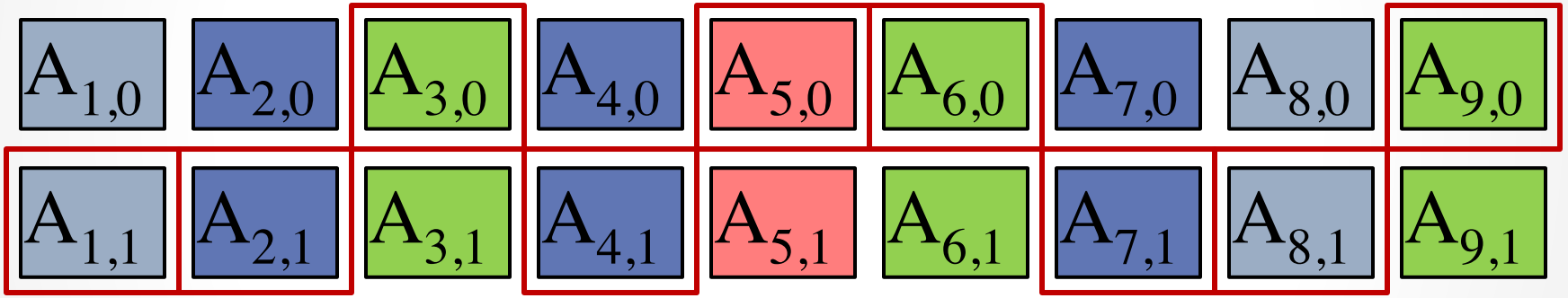
- MIFE
  - Order revealing encryption (ORE)
- Obfuscation
  - VBB point function obfuscation

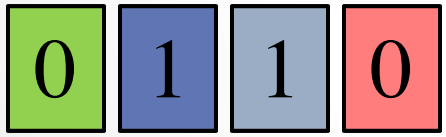
Applications



- MIFE
  - Order revealing encryption (ORE)
- **Obfuscation**
  - **VBB point function obfuscation**

Branching Programs

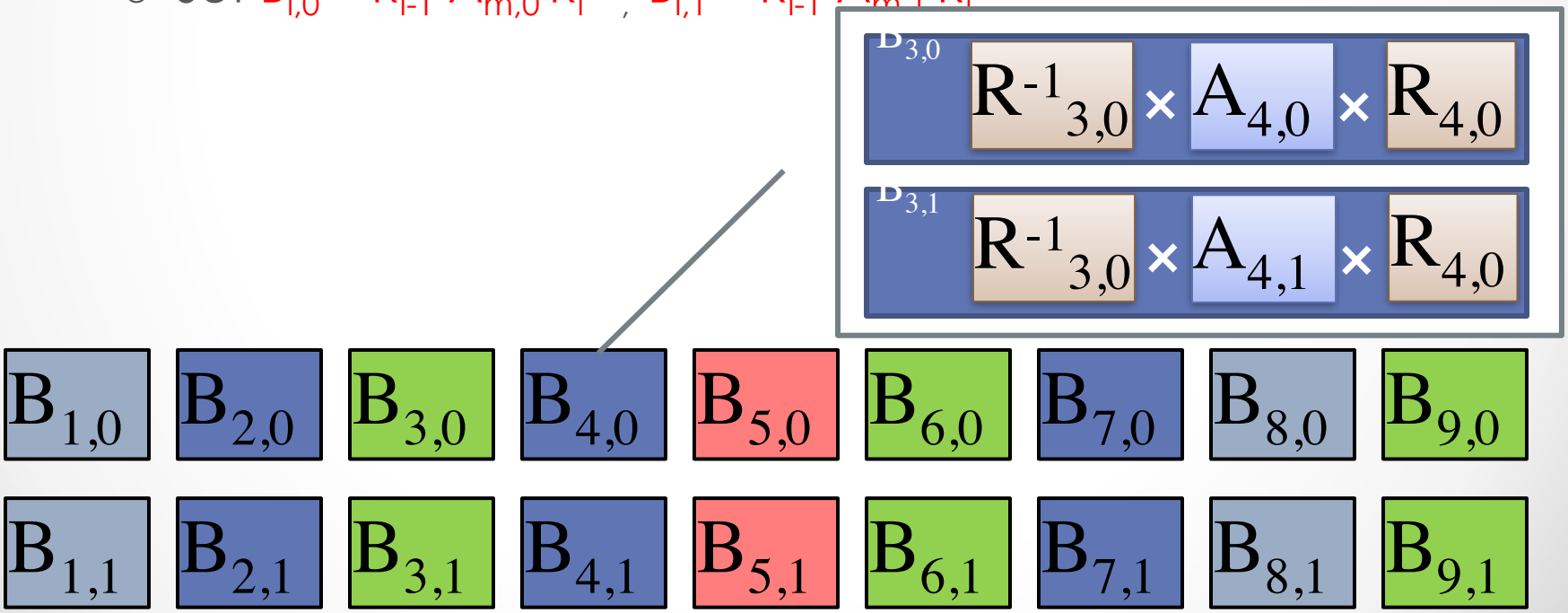




- Multiply the chosen 9 matrices.
  - If the product is I, output 1. Otherwise, output 0.

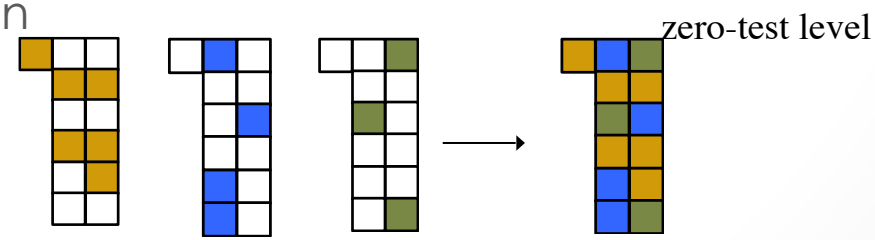
Obfuscation

- Randomized branching programs(RBP) [Kilian88]:
  - BP: $(inp(1), A_{1,0}, A_{1,1}), \ldots, (inp(m), A_{m,0}, A_{m,1})$
  - Sample invertible matrices $R_0, \ldots, R_m \in \{0,1\}^{k \times k}$
  - Set $B_{i,0} = R_{i-1} A_{m,0} R_i^{-1}$, $B_{i,1} = R_{i-1} A_{m,1} R_i^{-1}$



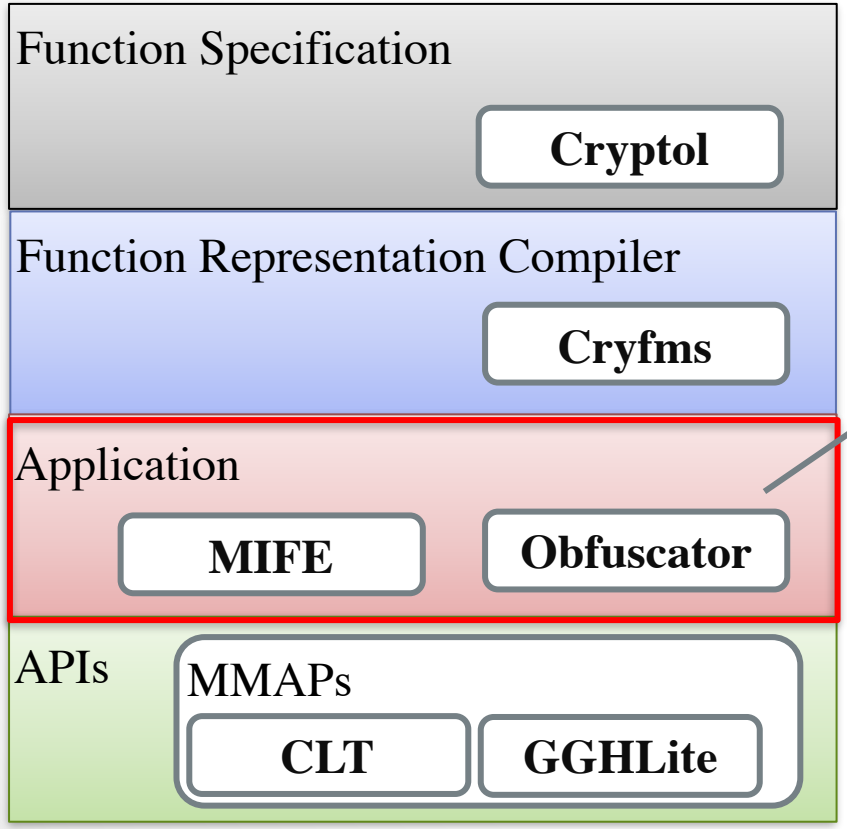
Obfuscation

- Multilinear maps (mmaps) [BS02]: $[a]_S$
  - $[a]_S + [b]_S = [a+b]_S$
  - $[a]_S + [b]_T = [ab]_{S \cup T}$
  - Zero test for level U – **is $[a]_U$ an encoding of 0?**
  - Candidate constructions and attacks
- Encode the randomized branching program matrices (each of their entries) with multilinear maps (mmaps)
  - Straddling encoding level sets – enforce input consistency, prevent partial evaluation



- Evaluate and zero-test

Applications

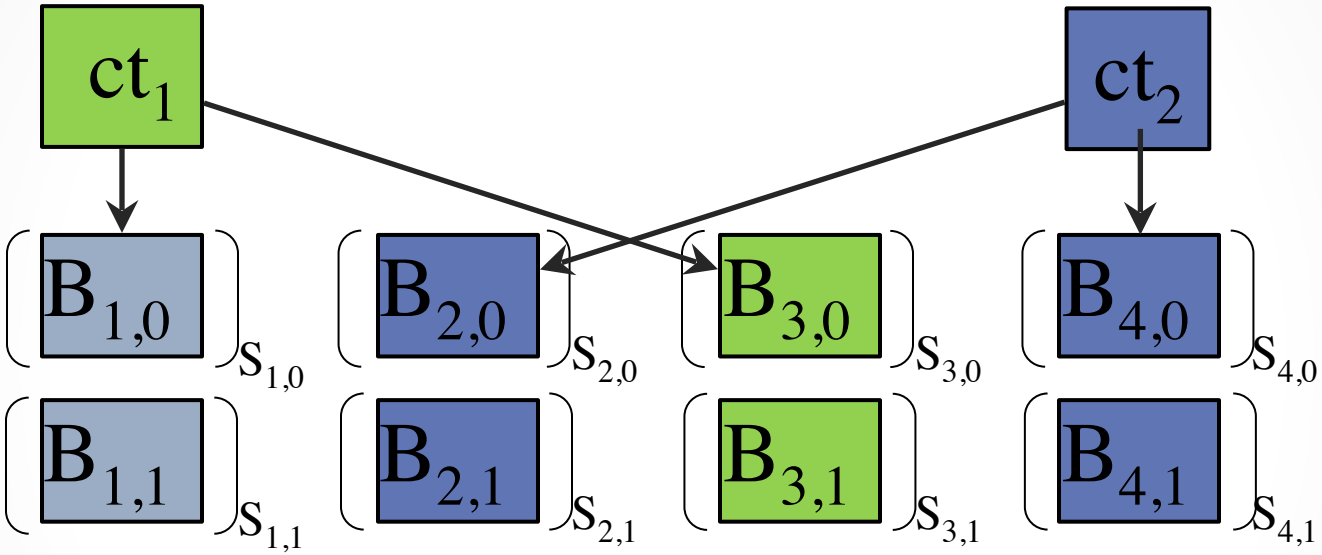


- **MIFE**
  - **Order revealing encryption (ORE)**
- Obfuscation
  - VBB point function obfuscation

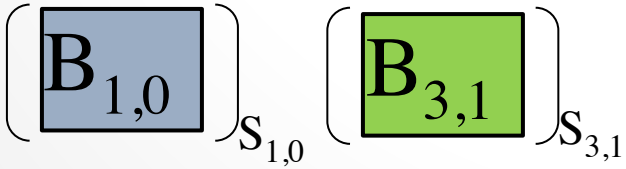
Private Single Key MIFE

- Setup – generate:
  - Master secret key MSK
  - Decryption key SK_F for multi-input function $F(x_1, \ldots, x_n)$
- Encryption
  - $ct^{(i)} = Enc^{(i)}(MSK, x_i)$
- Decryption
  - $Dec(SK_F, ct^{(1)}, \ldots, ct^{(n)}) = F(x_1, \ldots, x_n)$

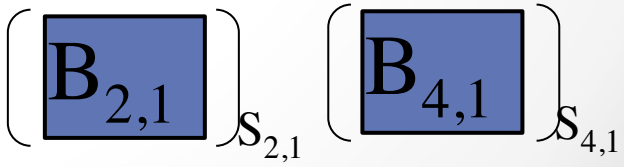
MIFE Construction [BLRSZZ15]



$\mathrm{Enc}^{(1)}(01)$:



$\mathrm{Enc}^{(2)}(11)$:



Mix-and-Match Attacks

$\mathrm{Enc}^{(1)}(01)$: $\left[B_{1,0} \right]_{S_{1,0}} \left[B_{3,1} \right]_{S_{3,1}}$

$\mathrm{Enc}^{(1)}(10)$: $\left[B_{1,1} \right]_{S_{1,1}} \left[B_{3,0} \right]_{S_{3,0}}$

$\mathrm{Enc}^{(1)}(11)$: $\left[B_{1,1} \right]_{S_{1,1}} \left[B_{3,1} \right]_{S_{3,1}}$

MIFE as a constrained formed of obfuscation

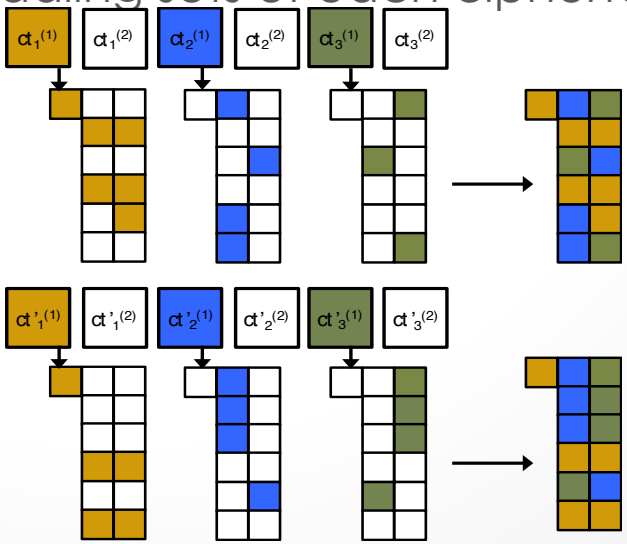
Exclusive Partition Families

- $(2^{\lambda},d)$- exclusive partition family;
  Let $\rho(i)$ be the binary representation of i
  - $U = (a_1, \ldots, a_d, b_{1,1}, \ldots, b_{2,\lambda}, \ldots, b_{d,1}, \ldots, b_{d,\lambda})$
  - $S_{i,1} = \{a_1\} \cup_{j \in [2,d]} \{b_{j,k} : \rho(i)[k] = 1\}$
  - $S_{i,j} = \{a_j\} \cup \{b_{j,k} : \rho(i)[k] = 0\}$

$i = 1001\ (9)$

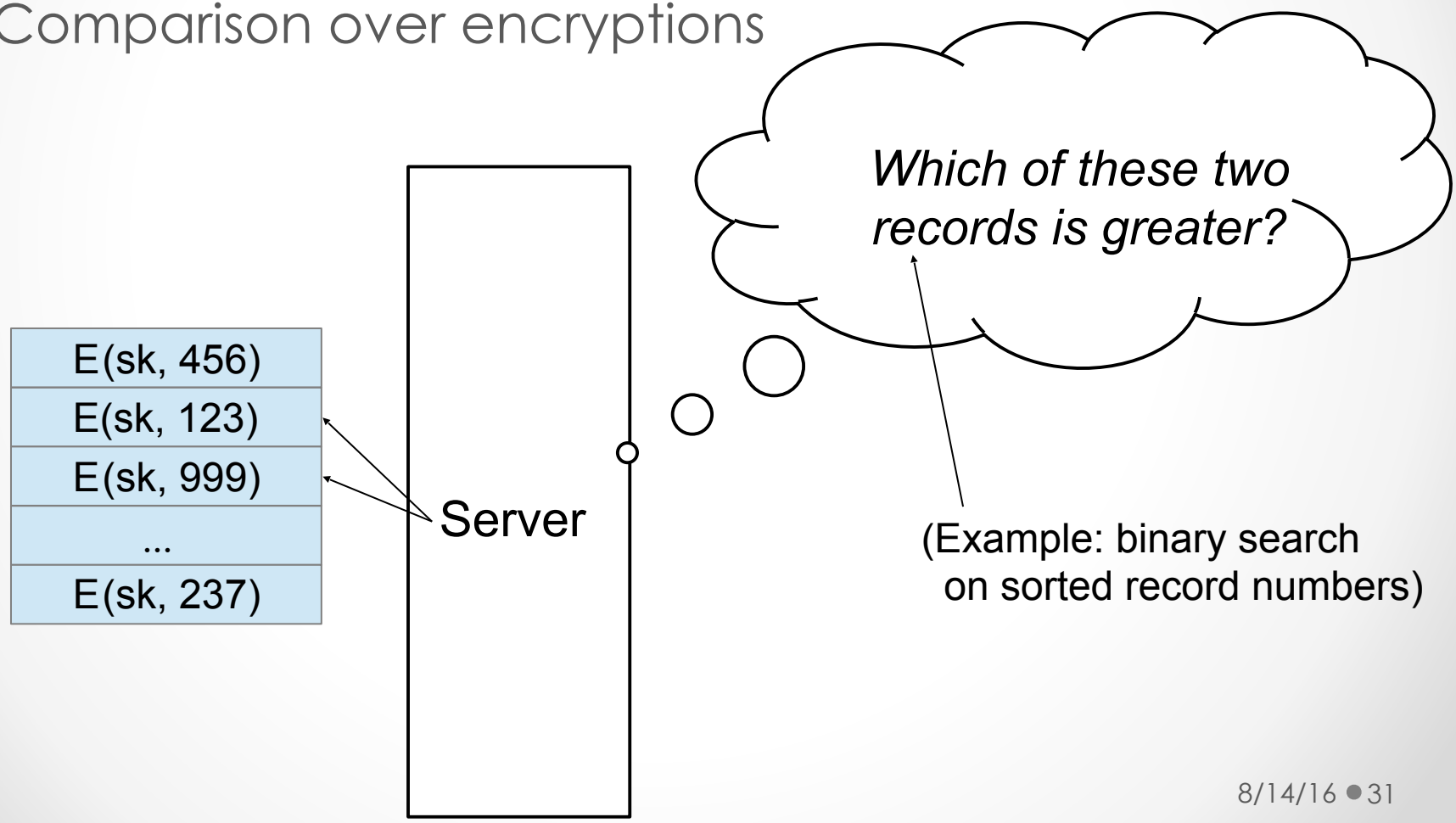
a_1	a_2	a_3	a_4
	$b_{2,1}$	$b_{3,1}$	$b_{4,1}$
	$b_{2,2}$	$b_{3,2}$	$b_{4,2}$
	$b_{2,3}$	$b_{3,3}$	$b_{4,3}$
	$b_{2,4}$	$b_{3,4}$	$b_{4,4}$

- Use a partition sampled at random from the exponential size set $\{ (S_{i,1}, \ldots, S_{i,d}) \}_{i \in [1,2^{\lambda}]}$ for the straddling sets of each ciphertext



Order-Revealing Encryption(ORE)

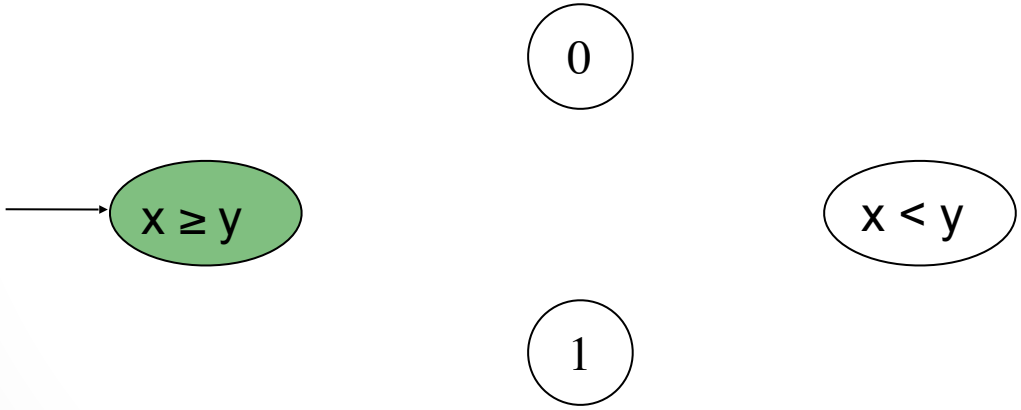
- Comparison over encryptions



Overview of Techniques

- View comparison function as (multi-input) finite automaton

Compare(x = 10, y = 11):



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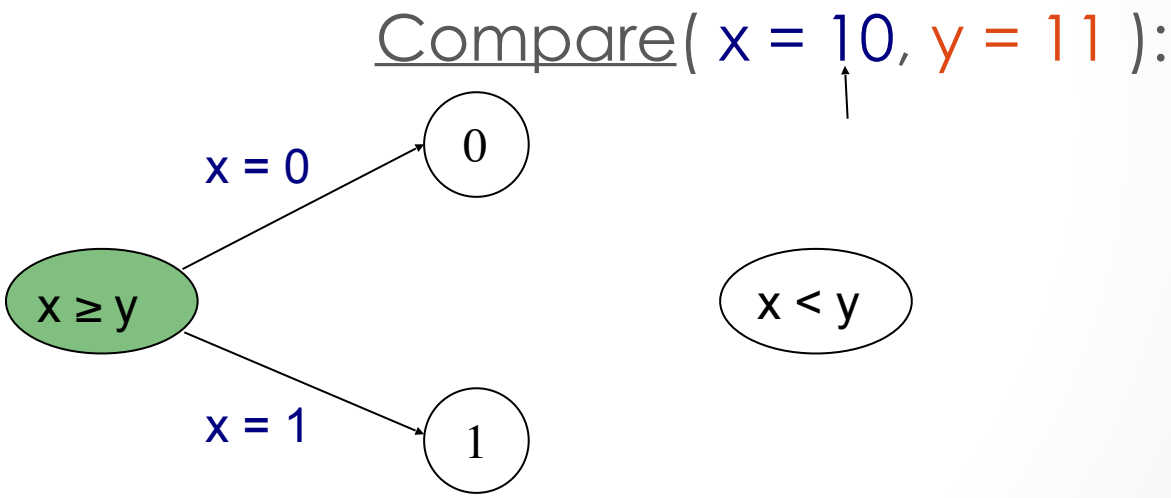

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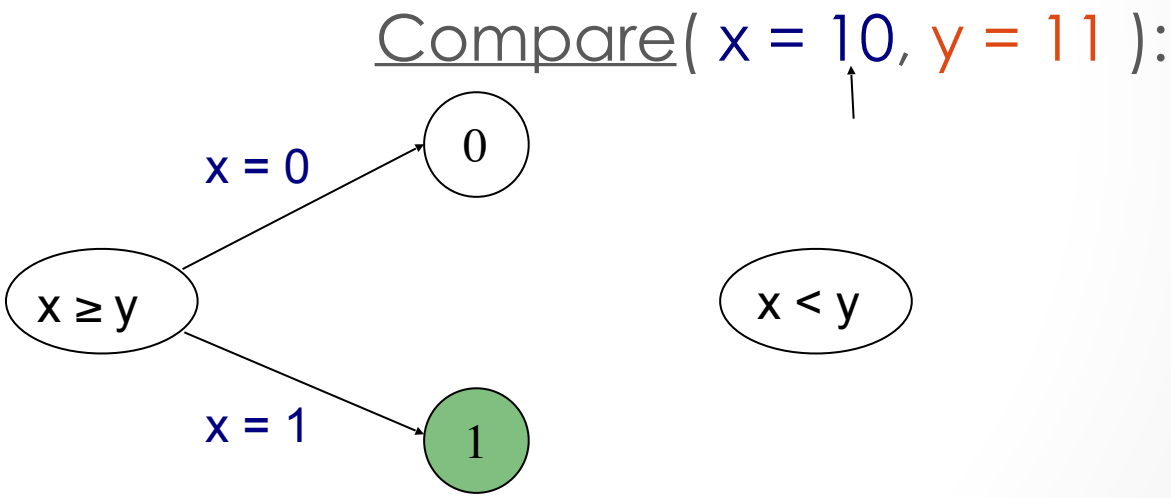
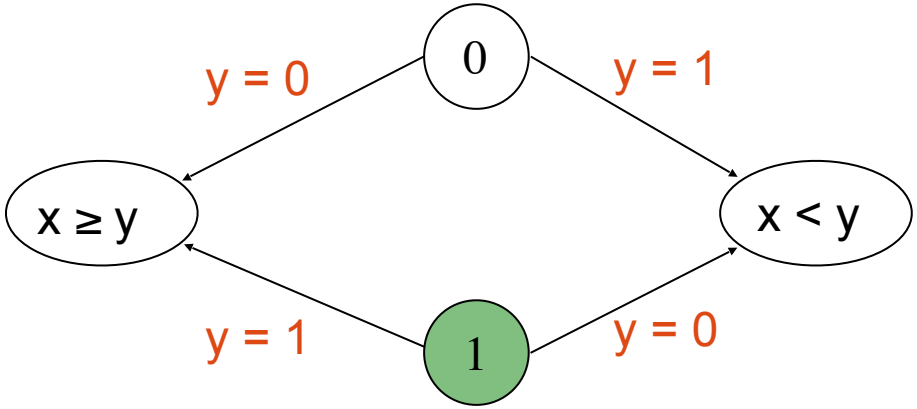


Overview of Techniques

- View comparison function as (multi-input) finite automaton

Compare(x = 10, y = 11):



Overview of Techniques

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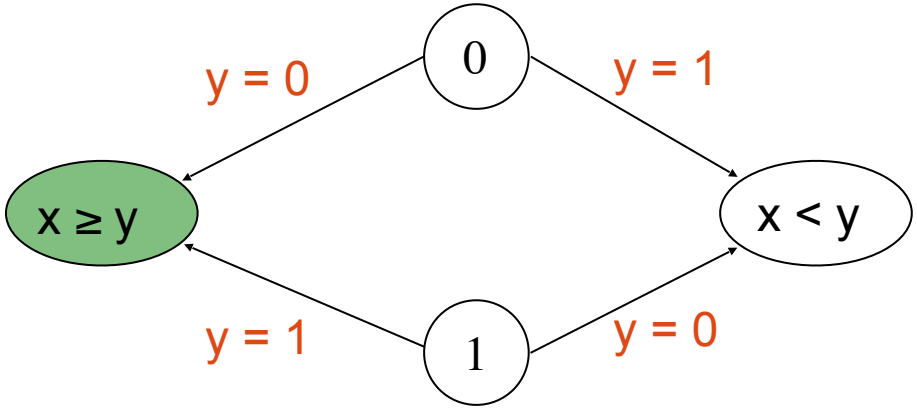
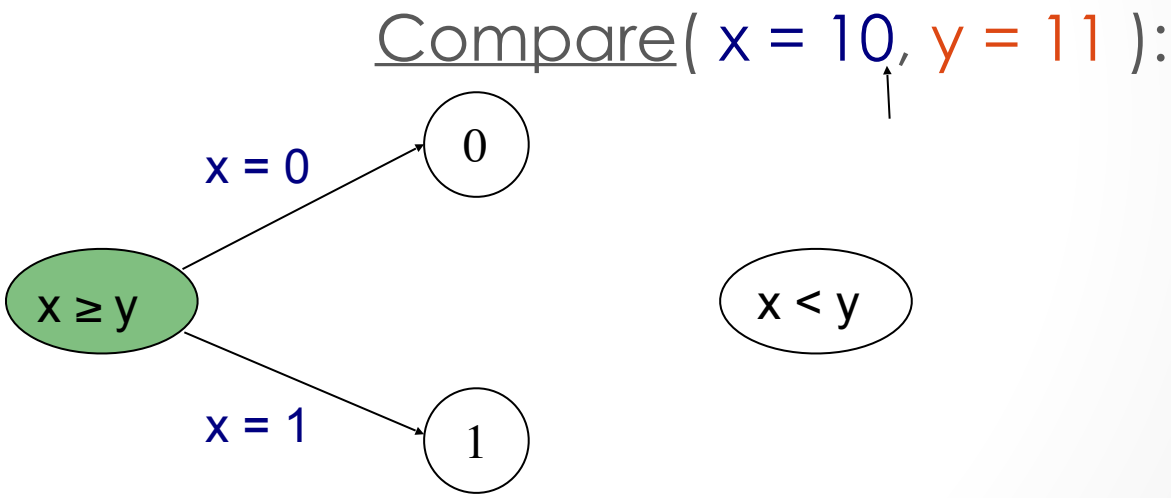
Compare(x = 10, y = 11):



Overview of Techniques

- View comparison function as (multi-input) finite automaton



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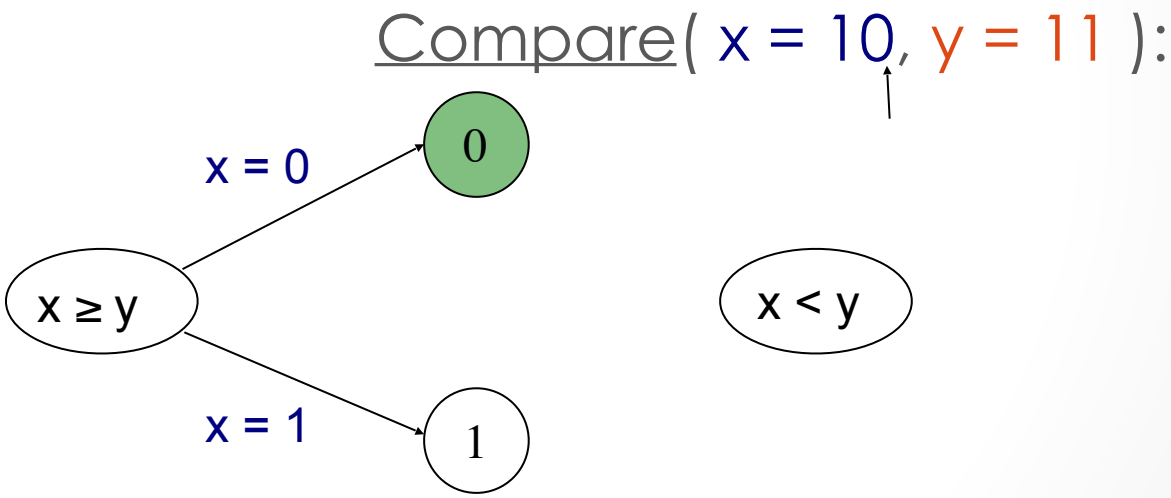

Overview of Techniques

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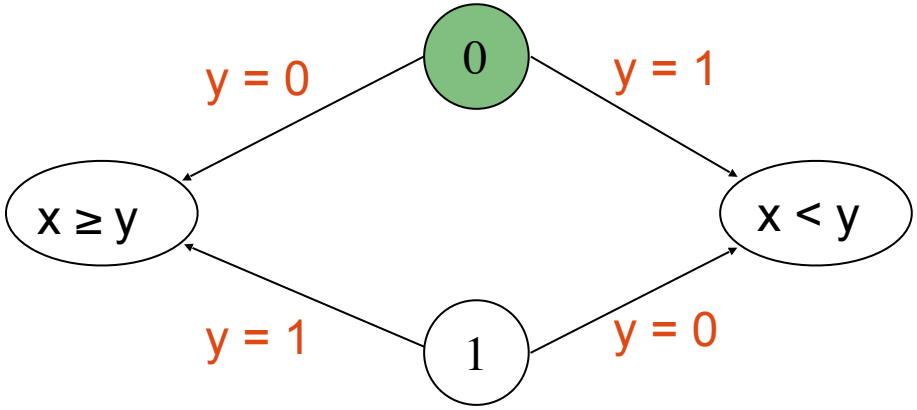
Compare(x = 10, y = 11):



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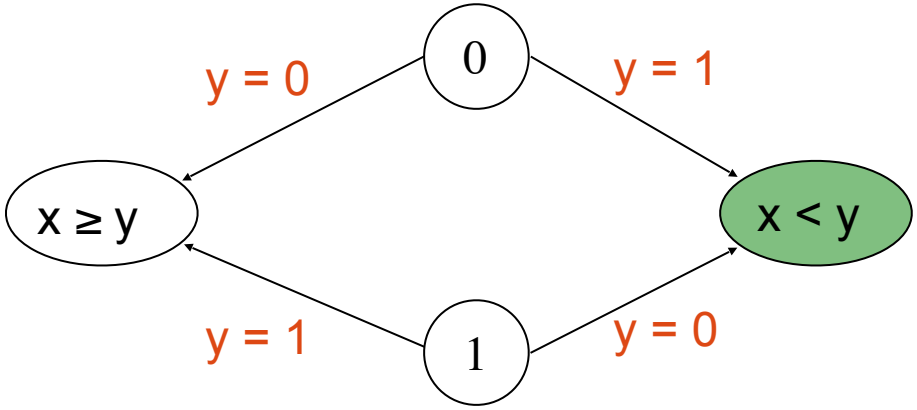


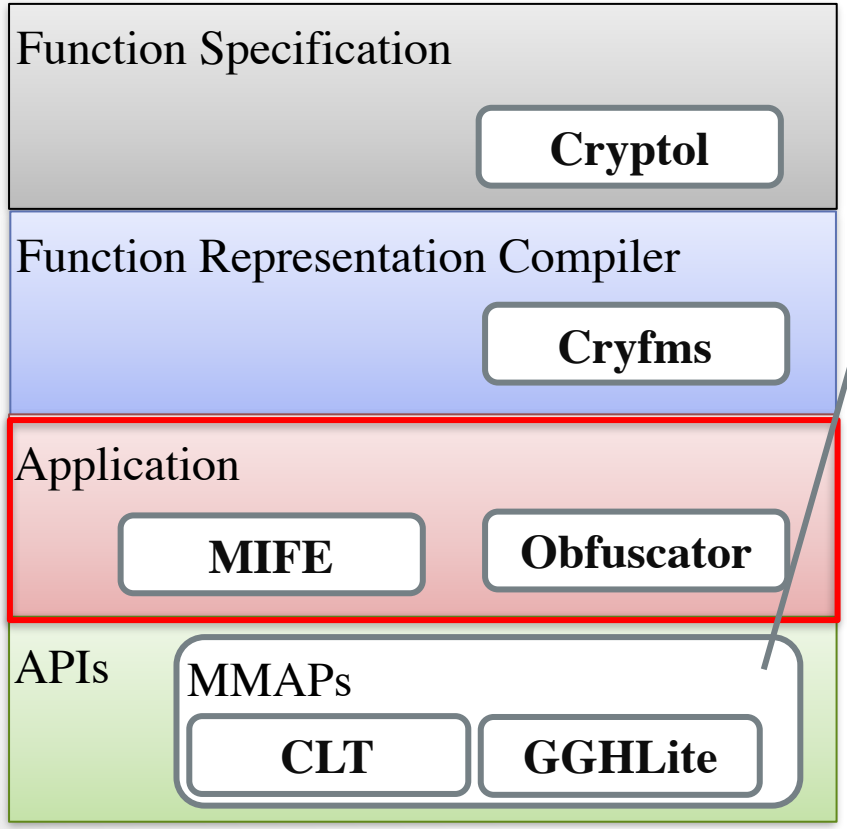
MIFE Instantiation

- 3-input DNF encryption

$$\mathsf{3DNF}(x, y, z) = (x_1 \wedge y_1 \wedge z_1) \vee \cdots \vee (x_n \wedge y_n \wedge z_n) \in \{0, 1\}$$

- Resemble “tribes” function
- Application: fraud detection

Applications



- MMAPs
  - GGHLite
  - CLT

vtable	function	comments
mmap_pp_vtable	fread/fwrite	read/write public parameters
	clear	clear public params
mmap_sk_vtable	init/clear	initialize/clear secret key
	fread/fwrite	read/write secret key
mmap_enc_vtable	init/clear	initialize/clear encoding
	fread/fwrite	read/write encoding
	set	copy encoding
	add	implements Add
	mul	implements Mult
	is_zero	implements ZeroTest
	encode	implements Encode

Multilinear Maps I

- **GGHLite**
  - Mmaps based on **ideal lattices**
  - A modification with parameter optimizations of [GGH13]
  - **Our implementation:**
    - Starting point: implementation of [ACLL15]
    - Extending capabilities of labelling encoding levels with respect to any subset of the zero-testing universe
    - Disk i/o for parameters
    - Parameter setting: follow [ACLL15]
  - *Attacks:* annihilation attacks [MSZ15]

Multilinear Maps II

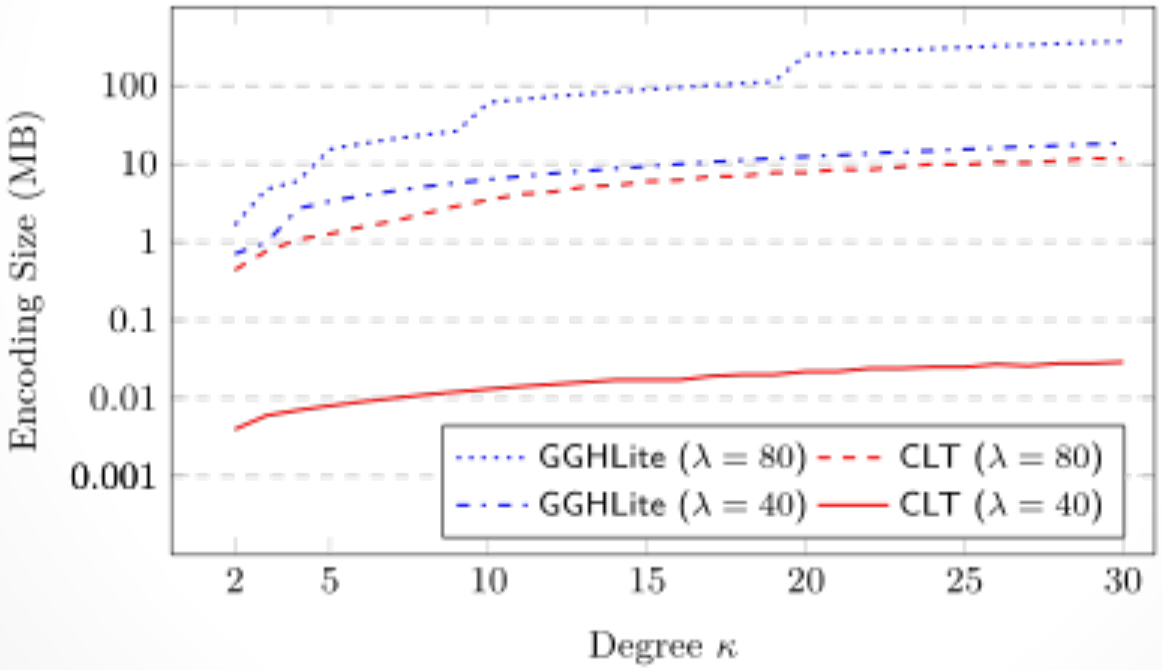
- **CLT**
  - Mmaps over the integers
  - [CLT13] without re-randomization public parameters
  - **Our implementation:**
    - Starting point implementation of [CLT13]
    - Encoding speedup - tree for computation of CRT
    - Disk i/o for parameters
    - Parameter setting: concrete estimates
  - *Attacks*: existing attacks with or without encodings of zero do not directly break our applications

Evaluation

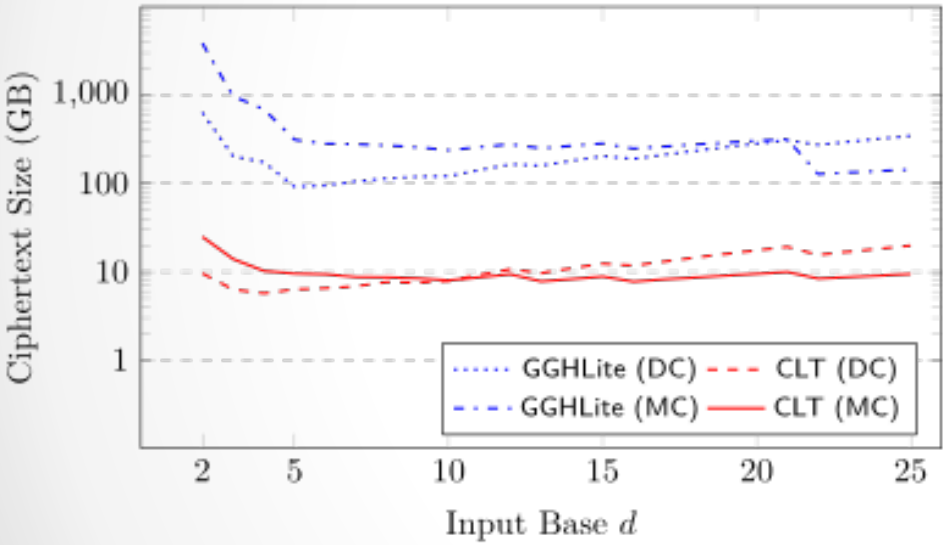
• • •

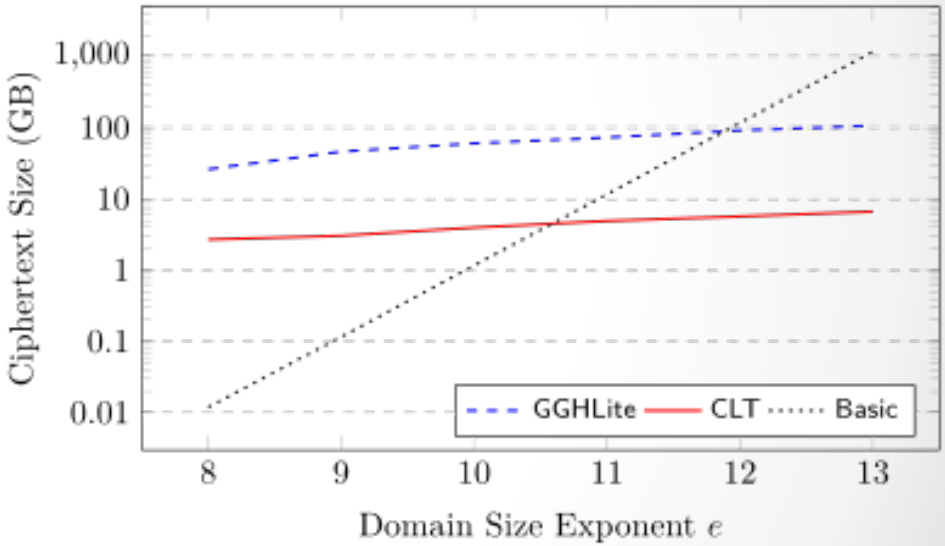
MMAP, ORE, 3DNF, Point Obfuscation

MMAP Encoding Sizes

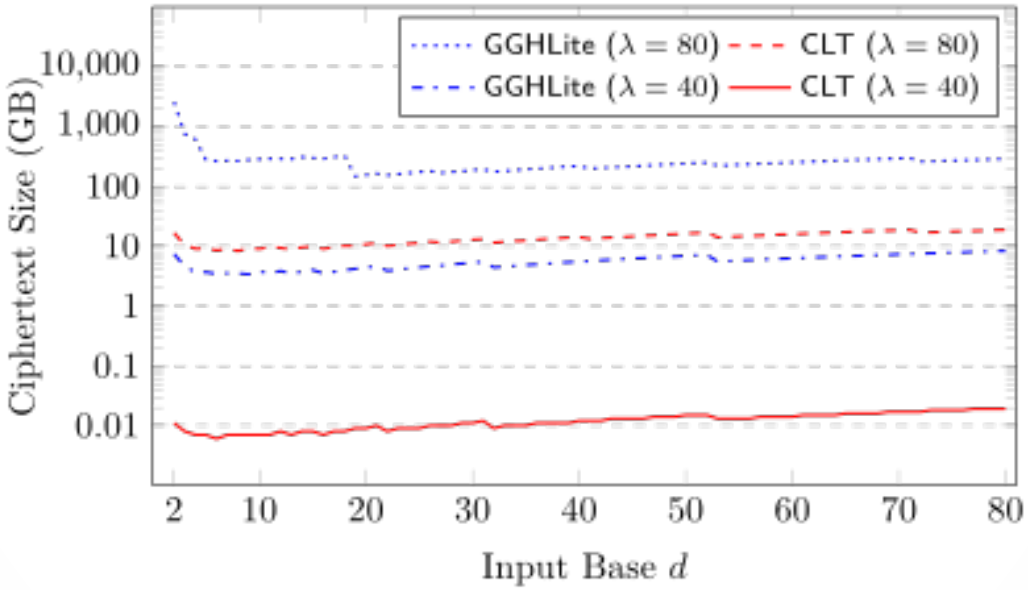


ORE Ciphertext Size





Obfuscation Size



Run-times

ORE

λ	experiment	encrypt	eval	|ct|	RAM
40	GGH (10^{12})	239 m	49 s	9.7 GB	8 GB
	GGH (10^{10})	198 m	43 s	6.4 GB	7 GB
	CLT (10^{12})	35 m	81 s	1.6 GB	20 GB
	CLT (10^{10})	24 m	54 s	1.0 GB	17 GB
80	GGH (10^{12})	151 h	8 m	92 GB	78 GB
	GGH (10^{10})	109 h	6 m	61 GB	72 GB
	CLT (10^{12})	489 m	8 m	7.3 GB	192 GB
	CLT (10^{10})	312 m	5 m	4.6 GB	166 GB

Obfuscation

λ	experiment	obf	eval	|obf|	RAM
40	GGH (40-bit)	1.8 h	6.8 s	3.6 GB	82 GB
	CLT (40-bit)	5.9 m	3.6 s	0.4 GB	2.7 GB
	GGH (80-bit)	3.0 h	29.7 s	13.7 GB	116 GB
	CLT (80-bit)	1.3 h	26.0 s	2.5 GB	20.5 GB
80	CLT (80-bit)	4.2 h	99.9 s	11.7 GB	227 GB

Conclusions

- **Framework for experimentation with mmaps and applications**
  - Modular – different parts can be used independently
- Evaluation of the efficiency of the state of the art

Thank you!