From Trivial Composition to Full Verification and an Application to Masking in Hardware







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executed operations





executed operations



executed operations



executed operations



executed operations

Masking (e.g., Boolean $x = x_0 + x_1 + \cdots + x_d$)



Noisy leakages security: $N \propto \frac{c}{MI(X;L)}$ Goal (ideally): $MI(X;L) < MI(X_i;L_i)^d$





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Probing security:

Sets of (d-1) probes are \bot of X (ideally)







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Security reductions



What can go wrong? (e.g., when computing a. b)

Issue #1. Lack of randomness (can break the independence assumption)

$$\begin{pmatrix} a_1b_1 & a_1b_2 & a_1b_3 \\ a_2b_1 & a_2b_2 & a_2b_3 \\ a_3b_1 & a_3b_2 & a_3b_3 \end{pmatrix} \Rightarrow \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

Example: probing $c_1 = a_1 \cdot (b_1 + b_2 + b_3)$ reveals information on b (when $c_1 = 1$)

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- mitigated by adding «refreshing gadgets »
- can be analyzed in the probing model

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Issue #2. Physical defaults

(can break the independence assumption)

Example: glitches (transcient values) « re-combine » the shares such that:

$$L_i = \delta(x_1 \cdot x_2 \cdot x_3)$$

(detected in the bounded moment model)



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Issue #2. Physical defaults

(can break the independence assumption)

- mitigated by adding a « noncompleteness » property
 [≈ Theshold Implementations]
- abstract property: can be analyzed in the probing model!





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- Solution #1: *direct verification* of (weaker) circuit properties
 - [Barthe et al., 2015/2019], [Bloem et al., 2018]
- Solution #2: *composable verification* with (stronger) properties
 - [Barthe et al., 2016] but limited to "abstract" circuits
- Solution #3: test more specific properties [Arribas et al., 2018]



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• Solution #1: *direct verification* of (weaker) circuit properties

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- Solution #2: composable verification with (stronger) properties
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- Can be complementary: use #1 for gadgets, #2 for circuits

Does it go wrong (for hardware masking)?

- State-of-the-art hardware-oriented masking schemes
 - Consolidating Masking Scheme (CMS, 2015)
 - Domain-Oriented Masking (DOM, 2016)
 - Unified Masking Approach (UMA, 2017)
 - Generic Low-Latency Masking (GLM, 2018)

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 - Theoretical concern or practical risk?
- [Moos et al., 2019]: all the higher-order extensions of these schemes are affected by concrete flaws
 - Next: CMS (local) and DOM (composability) examples...

Consolidating Masking Scheme

- Local flaw in the "ring refreshing" algorithm
 - Attack with 3 probes for any *d*>3 shares



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Fix proposed by De Cnudde $(\Rightarrow CMS \text{ more similar to DOM})$

Composability remains unclear



Composability requirements (example)



q-(Strong) Non Interference [Barthe et al., CCS 2016]: a circuit gadget (e.g., f1) is NI (SNI) any set of $q_1 + q_2$ probes can be simulated with at most $q_1 + q_2$ (only q_1) shares of each input

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Domain Oriented Masking

- Two algorithms: DOM-indep and DOM-dep
 - DOM-indep not sufficient to compose, e.g., $z = x \otimes x$

$$\begin{pmatrix} z_0 \\ z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} x_0 \otimes x_0 & \oplus & (x_0 \otimes x_1 \oplus r_0) & \oplus & (x_0 \otimes x_2 \oplus r_1) \\ (\mathbf{x_1} \otimes \mathbf{x_0} \oplus r_0) & \oplus & x_1 \otimes x_1 & \oplus & (x_1 \otimes x_2 \oplus r_2) \\ (\mathbf{x_2} \otimes x_0 \oplus r_1) & \oplus & (x_2 \otimes x_1 \oplus r_2) & \oplus & x_2 \otimes x_2 \end{pmatrix}$$

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 SOTA (2018): ∃ composable masking schemes that ignore physical defaults such as glitches & hardwareoriented masking schemes that mitigate glitches but are at best probing secure (so not provably composable)



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Example: p_1 gives a, b and c



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Definition: a gadget is **glitch-robust** *q***-SNI** if it is *q*-SNI in the "glitch-extended" probing model

⇒ Shares' fan in of secure gadgets should be minimum

 \Rightarrow Output probes (excluded in the SNI count) must be stable

Note: the problem must be solved jointly

• TI gadget + SNI refresh + register: robust against glitches & composable without glitches (not both)



• Extended probe on c' reveals all \mathcal{R} 's randomness

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• TI gadget + SNI refresh + register: robust against glitches & composable without glitches (not both)



- Extended probe on c' reveals all \mathcal{R} 's randomness
- Adding a register does not help (just probe c)






















DOM-indep is glitch-robust q-NI in 1 cycle



- [Faust et al., 2018]: glitch-robust SNI mult. in 2 cycles
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- Next: focus on trivial composition (natural first step & instrumental in our tool for full verification)

Improving trivial composition

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• But "share-by-share" linear gadgets are only NI



⇒ Trivial SNI composition must refresh linear gadgets

Probe Isolating Non-Interference (PINI)

• [Cassiers & Standaert, 2018]: gadgets should behave (w.r.t. simulatability) as if shares were isolated



⇒ "share-by-share" linear gadgets are PINI (formalizes the idea of circuit share in DOM/TIs)

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- Theorem: any combination of *q*-PINI gadgets is *q*-PINI
- Used to prove a strategy by [Goudarzi & Rivain, 2017]
- But can lead to much more...



• (∃ more efficient PINI multiplications in software)

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- Significantly improves trivial composition in hardware



[Faust et al., 2018]: SNI-based PINI mult. in 4 cycles

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[Cassiers et al., 2019]: PINI maintained without output register (...remember this would not work with SNI)

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 \approx optimization of [Faust et al., 2018] or fix of DOM

Other PINI advantages

- First efficient glitch-resistant masking scheme that is provably composable at arbitrary orders
 - Further improvements with optimized composition are an interesting open problem
 - But overheads compared to 1-cycle DOM limited
 - Especially for some S-box structures that can take advantage of the input refreshing asymmetry

Other PINI advantages

- First efficient glitch-resistant masking scheme that is provably composable at arbitrary orders
 - Further improvements with optimized composition are an interesting open problem
 - But overheads compared to 1-cycle DOM limited
 - Especially for some S-box structures that can take advantage of the input refreshing asymmetry
- Instrumental in the design of *full verification* tool
 - Composable verification like [Barthe et al., 2016] that applies to synthetized VHDL code rather than abstract (e.g., glitch-free) circuit descriptions
 - Also captures transitions (thanks to isolation)?

State-of-the-art tools (roughly)

	abstract	concrete
direct	Barthe et al. (Eurocrypt 2015)	REBECCA (Eurocrypt 2018) maskVerif (ESORICS 2019)
compbased	maskComp. (ACM CCS 2016) Tight Private Circuits (Asiacrypt 2018)	fullVerif (<i>new</i>)

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- I other approaches (e.g., spanning multiples cells like the one by Eldib et al., or aiming at different, more specific, goals like the one of Arribas et al.)
- Next, first full verification tool that applies to synthetized HDL code and captures all physical defaults that can be naturally modeled with probes (i.e., transitions & glitches)

Hardware composition verification tool

Trivial composition makes it simple for the designer:

"Just connect PINI gadgets together."

Do you really want to write a tool to check that all gadgets are PINI ?

for gadget in gadgets:
 assert gadget.is_pini(); // Uses maskVerif

Done?

A masked Verilog block cipher implementation

Code:

- ~30 files
- ~4k LoC

Parmeters:

- d = 2,...,16
- roll_sb = 0, ..., 5
- roll_lb = 0, 1, 2

15*6*3 = 270 parameter sets

Complex code:

- FSM
- loops
- procedurally generated code
- pipelining
- thousands of gadget instances
 - ...

Example LoC:

```
rinrfrs1_chunk[Nrndrfrs1_each-1+ii*Nrndrfrs1_each -: Nrndrfrs1_each]
<= {rinrfrs1[(Nrndrfrs1_each/4)*4-1+(ii+8)*Nrndrfrs1_each -:
(Nrndrfrs1_each/4)],{(Nrndrfrs1_each/4){1'b0}}, rinrfrs1[(Nrndrfrs1_each/4)*2-
1+(ii+8)*Nrndrfrs1_each -: (Nrndrfrs1_each/4)],{(Nrndrfrs1_each/4){1'b0}};</pre>
```

Is this thing (glitch, transition)-robust t-probing secure ?

What could go wrong ?

- Bad randomness input to gadgets
- Re-order of wires in a sharing
- Mix clock signals
- Mix valid and invalid data
- Output data at the right cycle
- Keep state around after computation is over

•

Code written by a side-channel expert hardware designer.

And no:

• Use non-PINI gadgets (Experiment might be biased.)

Tool workflow



- 2.5kLoC
- <10s runtime

Flexible: it is easy to implement other strategies.

Tool workflow



- 2.5kLoC
- <10s to check

Checking other strategies: 1 box change!

Source annotations

```
1 (* psim prop = "PINI", psim strat = "assumed", psim order=d *)
 2 module and_pini #(parameter d=2) (ina, inb, rnd, clk, out);
 3
 4 `include "ref rnd.inc"
 5 localparam n_rnd_mul = d*(d-1)/2;
6 localparam n_rnd = and_pini_nrnd;
8 (* psim_type = "sharing", psim_latency = 1 *) input [d-1:0] ina;
9 (* psim type = "sharing", psim latency = 0 *) input [d-1:0] inb;
10 (* psim type = "sharing", psim latency = 2 *) output [d-1:0] out;
11 (* psim type = "clock" *) input clk;
12 (*
      psim_type = "random", psim_count=4,
13
14
       psim rnd lat 0=1, psim rnd count 0=and pini lat 1,
       psim rnd lat 1=0, psim rnd count 1=and pini lat 0,
15
       psim rnd lat 2 = -1, psim rnd count 2=and pini lat m1,
16
       psim rnd lat 3=-2, psim rnd count 3=and pini lat m2
17
18 *) input [n rnd-1:0] rnd;
19
20 wire [n rnd mul-1:0] rnd mul = rnd[n rnd-1:ref n rnd];
21 wire [ref n rnd-1:0] rnd ref = rnd[ref n rnd-1:0];
22
23 wire [d-1:0] inb ref;
24 MSKref #(.d(d)) rfrsh (.in(inb), .clk(clk), .out(inb_ref), .rnd(rnd_ref));
25 MSKand #(.d(d)) mul (.ina(ina), .inb(inb_ref), .clk(clk), .rnd(rnd_mul), .out(out));
26
27 endmodule
```

Source annotations: composite gadget

```
1 (* psim prop = "PINI", psim strat = "composite", psim order=d *)
 2 module MSKspook_sbox #(parameter d=4) (in, rnd_ref, rnd_mul, clk, out);
 4 `include "spook sbox rnd.inc"
 5
6 (* psim_type = "sharing", psim_latency = 0, psim_count=spook_sbox_nbits*)
7 input [d*spook sbox nbits-1:0] in;
8 (* psim_type = "sharing", psim_latency = spook_sbox_lat, psim_count=spook_sbox_nbits *)
9 output [d*spook sbox nbits-1:0] out;
10 (* psim type = "clock" *) input clk;
11 (*
12 psim type = "random", psim count=6,
13 psim rnd lat 0=0, psim rnd count 0=2*ref lat 0,
14 psim_rnd_lat_1=-1, psim_rnd_count_1=2*ref_lat_m1,
15 psim rnd lat 2=-2, psim rnd count 2=2*ref lat m2,
16 psim_rnd_lat_3=1, psim_rnd_count_3=2*ref_lat_0,
17 psim_rnd_lat_4=0, psim_rnd_count_4=2*ref_lat_m1,
18 psim_rnd_lat_5=-1, psim_rnd_count_5=2*ref_lat_m2
19 *) input [4*ref n rnd-1:0] rnd ref;
20 (*
21 psim_type = "random", psim_count=2,
22 psim rnd lat 0=0, psim rnd count 0=2*and pini lat 1,
23 psim rnd lat 1=1, psim rnd count 1=2*and pini lat 1
24 *) input [4*and pini lat 1-1:0] rnd mul;
25
26 MSKreg #(d) reg1 (clk, 1'b0, in[d+d*(3)-1 -: d], x0F);
27 MSKreg #(d) reg2 (clk, 1'b0, in[d+d*(2)-1 -: d], x1F);
28 [...]
```
Source annotations: flatten (for the lazy)

```
1 (* psim_prop = "PINI", psim_strat = "composite", psim_order=d *)
 2 module MSKsbox_unit # (
       parameter d = 2,
 3
    parameter PDSBOX = 0,
 4
 5
     parameter Nbits = 128,
 6
     // Generation params (DO NOT TOUCH)
 7
    localparam AM BUND cols = 2**PDSBOX,
 8
      localparam SIZE BUND cols = d*Nbits/AM BUND cols,
      localparam AM cols = 32/AM BUND cols
 9
10 ) (cols, rnd, clk, cols_post_sb);
11 `include "spook sbox rnd.inc"
12 input [SIZE BUND cols-1:0] cols;
13 input [spook sbox rnd*(SIZE BUND cols/(4*d))-1:0] rnd;
14 input clk;
15 output [SIZE BUND cols-1:0] cols post sb;
16 genvar i:
17 for(i=0;i<AM cols;i=i+1) begin: sb
      MSKspook sbox #(.d(d)) sbi (
18
           .in(cols[(i+1)*4*d-1:i*4*d]),
19
           .rnd ref(rnd[ (i+1)*4*ref n rnd-1 +(32/(2**PDSBOX))*and pini lat 1
                                                                                    : i'
20
                                                                                    : i'
           .rnd mul(rnd[ (i+1)*4*and pini lat 1 -1
21
22
           .clk(clk),
           .out(cols post sb[(i+1)*4*d-1:i*4*d])
23
24
       );
25 end
26 endmodule
```

THANKS http://perso.uclouvain.be/fstandae/