A Physical Approach for Stochastic Modeling of TERO-based TRNG

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Random numbers in cryptography

- Random number generators constitute an essential part of (hardware) cryptographic modules
- The generated random numbers are used as:
  - Cryptographic keys (high security requirements)
  - Masks in countermeasures against side channel attacks
  - Initialization vectors, nonces, padding values, ...

Random numbers in logic devices

RANDOM NUMBER GENERATORS
(RNG)

DETERMINISTIC RNGs (DRNGs)

PHYSICAL TRUE RNGs (P-TRNGs)

DRNG + P-TRNG = Hybrid RNG

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Classical versus modern TRNG evaluation approach

- Two main security requirements on RNGs:
  - R1: Good statistical properties of the output bitstream
  - R2: Output unpredictability
- Classical approach:
  - Assess both requirements using statistical tests – often impossible
- Modern ways of assessing security:
  - Evaluate statistical parameters using statistical tests
  - Evaluate entropy using entropy estimator (stochastic model)
  - Test online the source of entropy using dedicated statistical tests

Our objectives

Propose a stochastic model of TERO-based TRNG [a]

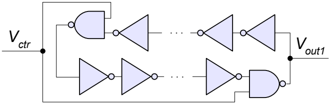
- Based on physical parameters quantifiable inside the device
- Can be used for online entropy assessment

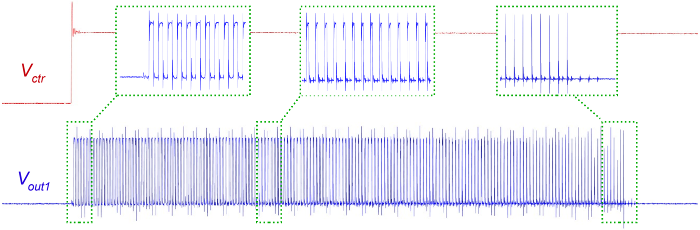
[a] M. Varchola and M. Drutarovsky, *New high entropy element for FPGA based true random number generators*, CHES 2010

Transition effect ring oscillator (TERO)

Principle:

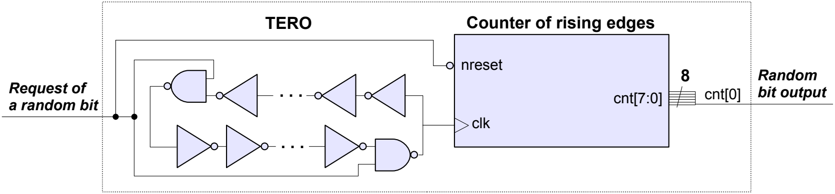
- Even number of inverters and two control gates in a loop
- Oscillates temporarily because of the difference in two branches
- Number of oscillations varies because of the intrinsic noise





TERO-based P-TRNG

Implementation:



- An asynchronous 8-bit counter counts random number of oscillations
- We use the counter to characterize the TERO
- The LSB of the counter (*cnt*(0)) is used also as the random bit (TRNG output)

Outlines of the modeling

Since the P-TRNG is periodically restarted, the counter values are mutually independent, therefore:

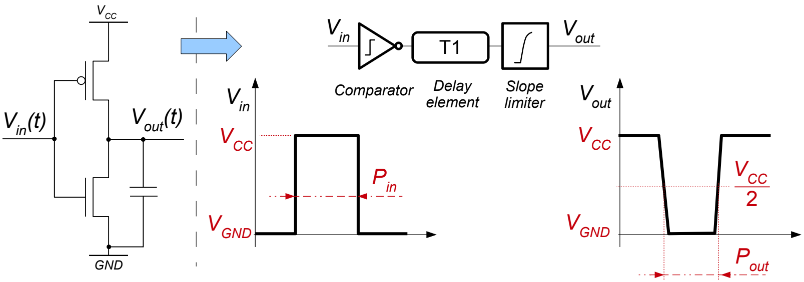
$$Entropy = -p_1 \cdot log_2(p_1) - (1 - p_1) \cdot log_2(1 - p_1),$$

where $p_1 = Pr\{cnt(0) = 1\}$.

We want to determine p_1, therefore, we need to analyze and characterize the distribution of counter values.

A noiseless inverter

Behavior of a noiseless inverter:

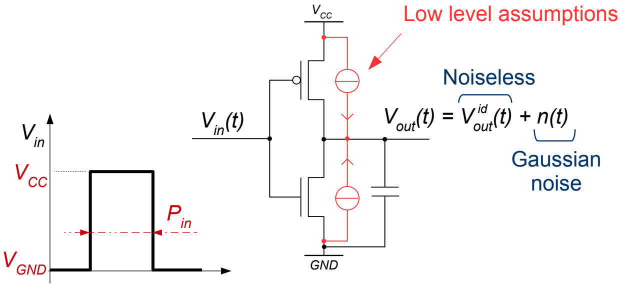


Analyzed by Reyneri *et al.*, [2] they determined $P_{out} = f(P_{in})$

[2] Reyneri *et al.*, *Oscillatory metastability in homogeneous and inhomogeneous flip-flops, IEEE SSC, 1990*

A noisy inverter

Behavior of a noisy inverter:

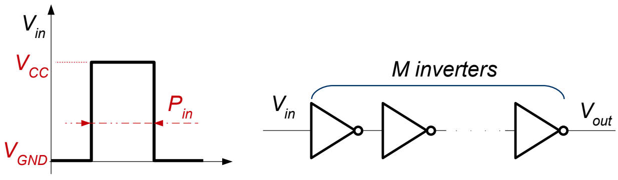


In the paper, using the model of Reyneri *et al.*, we determine

$P_{out} \sim \mathcal{N}(f(P_{in}), \sigma^2)$ (see *Lemma 1*)

An chain of M inverters

Impact of the noise on a chain of inverters:

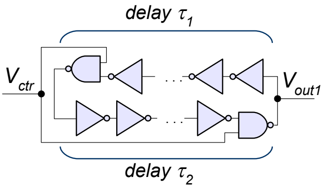


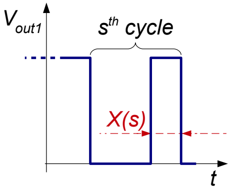
We apply *Lemma 1* to each inverter of the chain

We obtain $P_{out} \sim \mathcal{N}(F(P_{in}, M), G(\sigma^2, M))$

A loop of inverters

Impact of the noise on the duty cycle:





$$X(s) \sim N\left(\frac{\tau_1+\tau_2}{2} + \frac{\tau_2-\tau_1}{2}\cdot R^s,\ \sigma^2\cdot\frac{R^{2s+1}-1}{(1+H_d)^2-1}\right)$$

Geometric series (ratio R)

Stochastic model of TERO P-TRNG

The model characterizes distribution of counter values

- Objective: We want to get $Pr\{cnt = s\}$
- We just know the distribution of $X(s)$

We can use the equivalence $cnt > s \iff X(s) > 0$

Then

$$Pr\{cnt > s\} = \frac{1}{2}\left[1 - erf\left(K \cdot \frac{1 - R^{s-s_0}}{\sqrt{R^{2s+1} - 1}}\right)\right]$$

R is the ratio of the geometric series

K reflects the jitter σ^2

s_0 reflects the difference $\tau_1 - \tau_2$

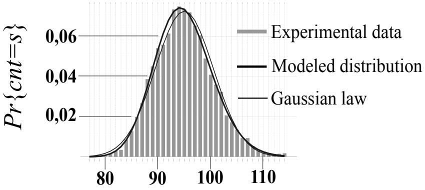
and

$$Pr\{cnt = s\} = Pr\{cnt \leq s\} - Pr\{cnt \leq s+1\}$$

Experimental validation

Validation of the modeled distribution using a χ^2 test

Experiment: TERO 1 in an ST Microelectronics 28 nm ASIC



$K = 35{,}680$
$s_0 = 94{,}152$
$R = 1{,}0153$

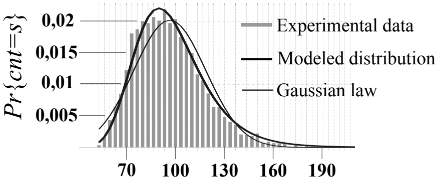
For a significance level $\alpha = 0.05$ and 38 degrees of freedom, the test statistic has to be lower than 53.384

Our model: the test statistic is 40.35
Gaussian law: the test statistic is 149.3

Experimental validation

Validation of the modeled distribution using a χ^2 test

Experiment: TERO 2 in an ST Microelectronics 28 nm ASIC



$K = 9{,}6939$
$s_0 = 90{,}675$
$R = 1{,}013$

For a significance level $\alpha = 0.05$ and 76 degrees of freedom, the test statistic has to be lower than 97.351

Our model: the test statistic is 33.97
Gaussian law: the test statistic is $> 10^6$

Entropy estimation

From our physical analysis we know $Pr\{cnt = s\}$
From $Pr\{cnt = s\}$ we compute $p_1 = Pr\{cnt(0) = 1\}$

Recall: Since the TERO is periodically restarted, the subsequent counter values are mutually independent and thus

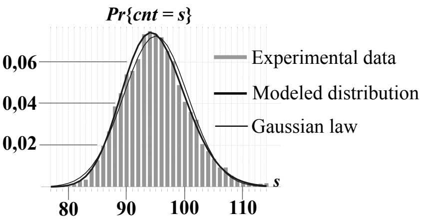
$$H_{sample} = -\sum_{s \in \mathbb{N}} p_s \log_2(p_s)$$

$$H_{lsb} = -p_1 \cdot log_2(p_1) - (1 - p_1) \cdot log_2(1 - p_1)$$

The second term represents the entropy of our TERO P-TRNG

Estimated entropy

Application of the model to TERO 1 and TERO 2



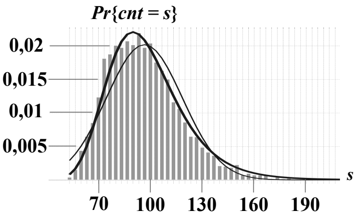
$K = 35{,}680$

$s_0 = 94{,}152$

$R = 1{,}0153$

$H_{sample} = 4{,}47$

$H_{lsb} > 0{,}999$



$K = 9{,}6939$

$s_0 = 90{,}675$

$R = 1{,}013$

$H_{sample} = 6{,}32$

$H_{cnt(0)} > 0{,}999$

- In the two cases the entropy of the raw binary signal exceeds the value 0.997 required by AIS31
- All generated bit streams passed tests T0 to T8 of AIS 31

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Conclusions

- We presented a **stochastic model** of the TERO P-TRNG
- The model is based on **transistor-level assumptions**
- The model was **validated** in an ASIC implemented using 28 nm ST Microelectronics technology
- We derived the **entropy** from this model
- The entropy and the output bit rate can be **easily managed** using the model

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